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SOME RELATIONS FOR HIGH PRESSURE FLOWS WITH AND WITHOUT HEAT TRANSFER

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Abstract—In the paper an analysis is made of oblique shock waves in a real gas flow. Expressions are obtained which describe the behaviour of the parameters such as pressure and temperatures in a shock wave. The expressions differ from the conventional ones by the fact that the present mathematical description of the effects in a shock wave involves various coefficients and quantities describing a real gas.

A method is presented for determination of correction factors before and after a shock wave. An expression is derived for a deflection angle. Then analytical expressions are obtained which define pressure, density and temperature discontinuities.

A calculation procedure is described.

NOMENCLATURE

w, gas velocity [m/s];
ρ, flow density [kg/m^3];
p, flow pressure [N/m^2];
v, specific gas volume [m^3/kg];
T, flow temperature [°K];
T_0, stagnation temperature [°K];
z, compressibility coefficient at flow parameters [p and T];
R, gas constant [J/kg.g];
x, adiabatic index of a real gas (a "temperature one");
c_p, mass heat capacity of a real gas at a constant pressure [J/kg . g];
c_v, mass heat capacity of a real gas at a constant volume [J/kg . g];
k, adiabatic index of a real gas;
$\bar{n}$, mean-integral value of an index ("volumetric") of an isentrope of a real gas;
a, sound velocity in a real gas [m/s];
y, sound velocity ratio in real and ideal gases (at a uniform temperature);
a_{cr}, critical velocity of a real gas [m/s];
ξ_{cr}, critical velocity ratio of real and ideal gases (at a uniform temperature T_0).

I. TOTAL IMPULSE OF A GAS FLOW

$$\mathcal{J} = \frac{G}{g}w + pF. \tag{1}$$

FOR THE flow in a cylindrical tube involving heat transfer it is possible to write

$$\mathcal{J} = \frac{G}{g}w_1 + p_1F = \frac{G}{g}w_2 + p_2F. \tag{2}$$

We have in [1]

$$a_{cr}^2 = \xi_{cr}^2 . 2\frac{k}{k+1}RT_0' \tag{3}$$

and

$$\frac{T_0'}{T} = \left[1 + \frac{1}{\bar{\mu}_T}\frac{x-1}{x}(z-z_0)_T\right].\left(1 - \frac{k}{k+1}\xi_{cr}^2\frac{1}{\bar{\mu}_T}\frac{x-1}{x}\lambda^2\right)^{-1} \tag{4}$$

where

$$\xi_{cr}^2 = y_{cr}^2 . \frac{k+1}{2}\left\{\frac{1}{\bar{\mu}_T}\frac{x-1}{x}\left[\frac{k}{2}y_{cr}^2 + (z_{cr} - z_0)T_{cr}\right] + 1\right\} \tag{5}$$

$$y_{cr}^2 = z_{cr}^2\left\{k\left[(\mu_T)_{cr} - \frac{x+1}{x}(\mu_p)_{cr}\right]\right\}^{-1} \tag{6}$$

$$\left.\begin{aligned} \mu_T &= -\frac{p^2}{RT}\left(\frac{\partial v}{\partial p}\right)_T \\ \mu_p &= \frac{p}{R}\left(\frac{\partial v}{\partial T}\right)_p \end{aligned}\right\} \tag{7}$$

$$x = \frac{c_p}{c_p - R\mu_p}. \tag{8}$$

Since $p/\rho = zRT$ and $w^2 = \lambda^2 a_{cr}^2$, then taking into account expressions (3) and (4) we have

$$\mathcal{I} = \frac{G}{g}w + pF =$$

$$\frac{G}{g}\left\{\lambda a_{cr} + \frac{k+1}{2k}\cdot\frac{z}{\xi_{cr}^2}\cdot a_{cr}\cdot\frac{1}{\lambda}\left(1 - \frac{k}{k+1}\cdot\frac{\xi_{cr}^2}{\bar{\mu}_T}\cdot\frac{x-1}{x}\cdot\lambda^2\right) \times\left[1 + \frac{1}{\bar{\mu}_T}\cdot\frac{x-1}{x}(z - z_0)_T\right]^{-1}\right\}. \tag{9}$$

For an ideal gas flow ($x = k$, $z = z_0 = \bar{\mu}_T = \xi_{cr} = 1$) expression (9) passes over to the known relation earlier obtained by B. M. Kiselev

$$\mathcal{I}_{ideal} = \frac{G}{g}w + pF = \frac{k+1}{2k}\cdot\frac{G}{g}a_{cr}\beta \tag{10}$$

where

$$\beta = \lambda_{ideal} + \frac{1}{\lambda_{ideal}}. \tag{11}$$

Equation (9) is of great importance in design problems on shock waves, on flows in the presence of heat transfer and in some other cases.

II. OBLIQUE ADIABATIC SHOCK WAVES IN HIGH-PRESSURE ZONES

In [1] we have

$$w_2^2 - w_1^2 = 2\left[R\bar{\mu}_T\frac{x-1}{x}(T_1 - T_2) - RT_1 (z_2 - z_1)_{T_1}\right]. \tag{12}$$

In a direct shock wave there takes place a discontinuity of a flow velocity from w_1 to w_2. In an oblique shock wave only a normal flow velocity component (w_n) from w_{1n} to w_{2n} is subjected to the discontinuity.

It goes without saying that $w_{2n} < w_{1n}$, and the tangential velocity component w_t in a shock wave does not vary ([2], Fig. 1).

Bearing in mind that $w_2^2 - w_1^2 = w_{2n}^2 - w_{1n}^2$ as well as the expressions following from the state equation

$$T_1 = \frac{p_1}{\rho_1 - z_1 R}$$

and

$$T_2 = \frac{P_2}{\rho_2 z_2 R},$$

equation (12) is reduced to the form

$$w_{1n}^2 - w_{2n}^2 = 2\frac{x}{x-1}\left\{\frac{\bar{\mu}_T}{z_2}\cdot\frac{p_2}{p_1} - \left[\frac{\bar{\mu}_T}{z_1} + \frac{1}{z_1}(z_1 - z_2)_{T_1}\frac{x-1}{x}\right]\frac{\rho_2}{\rho_1}\right\}\frac{p_1}{\rho_2}. \tag{13}$$

Using the momentum law in the form

$$p_2 - p_1 = \rho w(w_{1n} - w_{2n}) \tag{14}$$

and the continuity equation for an oblique shock wave

$$\rho_1 w_{1n} = \rho_2 w_{2n} \tag{15}$$

as a result of simultaneous solution of (13), (14) and (15) upon transformations we arrive at

$$\left(\frac{p_2}{p_1} - 1\right)\left(\frac{\rho_2}{\rho_1} + 1\right) = 2\frac{x}{x-1}\left\{\frac{\bar{\mu}_T}{z_2}\frac{p_2}{p_1} - \left[\frac{\bar{\mu}_T}{z_1} + \frac{1}{z_1}(z_1 - z_2)_{T_1} \times\frac{x-1}{x}\right]\frac{\rho_2}{\rho_1}\right\}. \tag{16}$$

Let us elucidate how ρ_2/ρ_1 depends on the velocity coefficient up to a shock wave λ_{1n} in an oblique shock wave for a real gas flow.

Neglecting heat transfer during a shock wave, i.e. assuming T_0 constant, write down expression (4) in the form

$$T_0 = T_1(1 + b_1) + \frac{k+1}{kR} \cdot \frac{c_1}{\xi_{1cr}^2} \cdot \frac{w_1^2}{2} =$$

$$T_2(1 + b_2) + \frac{k+1}{kR} \cdot \frac{c_2}{\xi_{2cr}^2} \cdot \frac{w_2^2}{2} \quad (17)$$

where

$$\left.\begin{aligned} b &= \frac{1}{\bar{\mu}_T} \cdot \frac{x-1}{x}(z - z_0)_T \\ c &= \xi_{cr}^2 \cdot \frac{k}{k+1} \cdot \frac{1}{\bar{\mu}_T} \cdot \frac{x-1}{x}. \end{aligned}\right\} \quad (18)$$

According to the kinetics of the flow (Fig. 1)

$$\left.\begin{aligned} w_1^2 &= w_{1n}^2 + w_t^2 \\ w_2^2 &= w_{2n}^2 + w_t^2. \end{aligned}\right\} \quad (19)$$

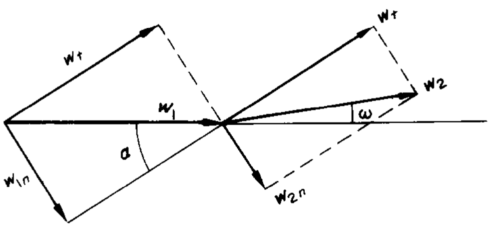

FIG. 1.

Taking into account that

$$\frac{c_1}{\xi_{1cr}^2} = \frac{c_2}{\xi_{2cr}^2} = \frac{c}{\xi_{cr}^2} = \frac{1}{\bar{\mu}_T} \frac{k}{k+1} \frac{x-1}{x},$$

as well as regarding for expression (19) upon a number of transformations we have

$$a_{cr}^2 = w_{1n} \cdot w_{2n} + \xi_{cr}^2 \cdot \frac{1}{\bar{\mu}_T} \cdot \frac{k}{k+1} \cdot \frac{x-1}{x} \cdot w_t^2. \quad (20)$$

Equation (20) relates a critical velocity to the product of velocities before and after an oblique shock wave in a real gas flow.

It is possible to write (Fig. 1)

$$\left.\begin{aligned} w_{1n} &= w_1 \cdot \sin\alpha \\ w_{2n} &= w_2 \cdot \sin\beta \\ w_t &= w_1 \cos\alpha = w_2 \cos\beta \\ \beta &= \alpha - \omega. \end{aligned}\right\} \quad (21)$$

The velocity a_{cr} is averaged.

Take the geometric mean relation for a_{cr}, i.e. assume that

$$a_{cr} = \sqrt{(a_{1cr} \cdot a_{2cr})} \quad (22)$$

consequently

$$\xi_{cr}^2 = \xi_{1cr} \cdot \xi_{2cr}. \quad (23)$$

By analogy with that presented for a direct shock wave [4] it is possible to write

$$w_{1n} \cdot w_{2n} = \frac{p_2 - p_1}{\rho_2 - \rho_1}. \quad (24)$$

Solving simultaneously equations (20)–(24) upon transformations we obtain

$$\frac{\rho_2}{\rho_1} = \frac{\xi_{1cr}}{\xi_{2cr}} \times \frac{\lambda_1^2 \cdot \sin^2\alpha}{1 - \xi_{1cr}^2(1/\bar{\mu}_T)(k/k+1)(x-1)/x \cdot \lambda_1^2 \cos^2\alpha}. \quad (25)$$

Passing again to equation (16) and taking into account equation (25) upon transformations, we arrive at

$$\frac{p_2}{p_1} = \left[\left[\left\{2\frac{x}{z_1}\left[\bar{\mu}_T + (z_1 - z_2)_{T_1} \cdot \frac{x-1}{x}\right] - x + 1\right\} \times (x-1)^{-1} \cdot \lambda_1^2 \cdot \sin^2\alpha \cdot \left[\frac{\xi_{2cr}}{\xi_{1cr}} \times \left(1 - \xi_{1cr} \cdot \frac{1}{\bar{\mu}_T} \cdot \frac{k}{k+1} \cdot \frac{x-1}{x} \times \lambda_1^2 \cdot \cos^2\alpha\right)\right]^{-1} - 1\right]\right] \cdot \left\{\left[x\left(2\frac{\bar{\mu}_T}{z_2} - 1\right) + 1\right]\right.$$

$$\times (x-1)^{-1} - \lambda_1^2 \sin^2\alpha \left[\frac{\xi_{2cr}}{\xi_{1cr}} \left[1 - \xi_{1cr} \times \frac{1}{\bar{\mu}_T} \frac{k}{k+1} \frac{x-1}{x} \cdot \lambda_1^2 \cos^2\alpha \right) \right]^{-1} \right\}^{-1}. \tag{26}$$

Since

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \cdot \frac{\rho_1}{\rho_2} \cdot \frac{z_1}{z_2}$$

then

$$\frac{T_2}{T_1} = \frac{z_1}{z_2} \left[\left[\left\{ 2 \frac{x}{z_1} \left[\bar{\mu}_T + (z_1 - z_2)_{T_1} \frac{x-1}{x} \right] - x + 1 \right\} \times (x-1)^{-1} - \frac{1}{\lambda_1^2 \sin^2\alpha} \cdot \frac{\xi_{2cr}}{\xi_{1cr}} \left(1 - \xi_{1cr} \cdot \frac{1}{\bar{\mu}_T} \times \frac{k}{k+1} \cdot \frac{x-1}{x} \cdot \lambda_1^2 \cos^2\alpha \right) \right]\right] \cdot \left\{ \left[x \left(2 \frac{\bar{\mu}_T}{z_2} - 1 \right) + 1 \right] (x-1)^{-1} - \lambda_1^2 \sin^2\alpha \left[\frac{\xi_{2cr}}{\xi_{1cr}} \times \left(1 - \xi_{1cr} \frac{1}{\bar{\mu}_T} \cdot \frac{k}{k+1} \cdot \frac{x-1}{x} \lambda_1^2 \cos^2\alpha \right) \right]^{-1} \right\}^{-1}. \tag{27}$$

Relations (25)–(27) are the main ones which describe a change in appropriate parameters (densities, pressures and temperatures) in an oblique shock wave in a real gas flow.

At $\alpha = 90°$ we arrive at the expression for a direct shock wave

$$\left(\frac{\rho_2}{\rho_1} \right)_{\text{direct}} = \frac{\xi_{1cr}}{\xi_{2cr}} \cdot \lambda_1^2 \tag{28}$$

$$\left(\frac{p_2}{p_1} \right)_{\text{direct}} = \left[\left[\left\{ 2 \frac{x}{z_1} \left[(\bar{\mu}_T + (z_1 - z_2))_{T_1} \frac{x-1}{x} \right] - x + 1 \right\} (x-1)^{-1} \cdot \lambda_1^2 \cdot \frac{\xi_{1cr}}{\xi_{2cr}} - 1 \right]\right] \times \left\{ \left[x \left(2 \frac{\bar{\mu}_T}{z_2} - 1 \right) + 1 \right] (x-1)^{-1} - \lambda_1^2 \times \frac{\xi_{1cr}}{\xi_{2cr}} \right\}^{-1} \tag{29}$$

$$\left(\frac{T_2}{T_1} \right)_{\text{direct}} = \frac{z_1}{z_2} \left[\left[\left\{ 2 \frac{x}{z_1} \left[\bar{\mu}_T + (z_1 - z_2)_{T_1} \times \frac{x-1}{x} \right] - x + 1 \right\} (x-1)^{-1} - \frac{1}{\lambda_1^2} \frac{\xi_{2cr}}{\xi_{1cr}} \right]\right] \times \left\{ \left[x \left(2 \frac{\bar{\mu}_T}{z_2} - 1 \right) + 1 \right] (x-1)^{-1} - \lambda_1^2 \times \frac{\xi_{1cr}}{\xi_{2cr}} \right\}^{-1}. \tag{30}$$

Relations (28)–(30) are obtained as a result of further precision of the expressions obtained [4] for the ratio of the appropriate parameters in a direct shock wave. In [4] it was assumed that $\xi_{cr} \approx \xi_{2cr} \approx \xi_{cr}$, i.e. a critical velocity of a real gas in a shock wave is assumed to be nearly constant while in precise equations (28)–(30) the critical velocity is a variable since the coefficient ξ_{cr} is a function of a pressure and temperature.

On the basis of equations (3), (20)–(23) we arrive at

$$\lambda_2^2 \cdot \frac{\xi_{2cr}}{\xi_{1cr}} = \left(1 - \frac{k}{k+1} \cdot \xi_{1cr}^2 \cdot \frac{1}{\bar{\mu}_T} \cdot \frac{x-1}{x} \times \lambda_1^2 \cos^2\alpha \right)^2 \cdot \left(\lambda_1^2 \cdot \frac{\xi_{1cr}}{\xi_{2cr}} \cdot \sin^2\alpha \right)^{-1} + \lambda_1^2 \cdot \frac{\xi_{1cr}}{\xi_{2cr}} \cdot \cos\alpha. \tag{31}$$

Hence at $\alpha = 90°$, i.e. for a direct shock wave in a real gas flow we have

$$\lambda_1 \cdot \lambda_2 = 1. \tag{32}$$

In [4] it has been shown that if the adiabatic curve equation in the form

$$pv^{\bar{n}} = \text{const} \tag{33}$$

is applied to the real gas, then for a mean integral "volumetric" index of an adiabatic curve for a real gas the expression is obtained in the form:

$$\bar{n}_{(1-2)} = \frac{1}{2}\left\{z_1\left[(\mu_T)_1 - \frac{x-1}{x}(\mu_p)_1\right]^{-1} + z_2\right.$$

$$\left.\times\left[(\mu_T)_2 - \frac{x-1}{x}(\mu_p)_2\right]^{1}\right\} \quad (34)$$

where μ_T and μ_p are expressed by equation (7).

For the ratio ξ_{cr}/ξ_{2cr} upon a number of the appropriate derivations the following expression is obtained:

$$\frac{\xi_{1cr}}{\xi_{2cr}} = \frac{1}{\lambda_1^2}\cdot\frac{\bar{n}+1}{\bar{n}-1}\left[1 - \left(\frac{T_1}{T_0}\right)^{\frac{x}{x-1}\frac{n-1}{n}}\right] \quad (35)$$

where $\bar{n}$ in this case is taken for the interval (0–1).

With a sufficient accuracy when determining the ratio ξ_{cr}/ξ_{2cr} the values of T_0 and λ_1 may be calculated from the relation for an ideal gas

$$T_0 \approx T_1 + \frac{k-1}{2kR}w_1^2 \quad (36)$$

and

$$\lambda_1 \approx \frac{k+1}{k-1}\left(1 - \frac{T_1}{T_0}\right). \quad (37)$$

To find the coefficients ξ_{1cr} and ξ_{2cr} by the known ratio ξ_{1cr}/ξ_{2cr} it is possible to use equation (23) if ξ_{cr}^2 is preliminarily determined.

It is easy to derive a convenient expression for finding ξ_{cr}^2.

Relation (3) conformably to the adiabatic flow (when T_0 is constant) is presented as:

$$a_{cr}^2 = \xi_{cr}^2 \,.\, 2\frac{k}{k+1}RT_0. \quad (38)$$

We have obtained [6]

$$a^2 = y^2 kRT \quad (39)$$

or

$$a_{cr}^2 = y_{cr}^2 kRT_{cr}. \quad (40)$$

Solving simultaneously equations (38) and (40) we arrive at

$$\xi_{cr}^2 = y_{cr}^2\frac{k+1}{2}\frac{T_{cr}}{T_0} \quad (41)$$

where y_{cr} is expressed by equation (6).

The relationship between the angles β and α is obtained in the form:

$$\operatorname{tg}\beta = \frac{x-1}{k+1}\xi_{cr}^2\left\{\frac{k}{x\bar{\mu}_T} + \frac{1}{y_1^2}\left[1 + \frac{1}{\bar{\mu}_T}\frac{x-1}{x}\right.\right.$$

$$\left.\left.\times(z_1 - z_0)_{T_1}\right]\cdot\frac{2}{x-1}\frac{1}{M_1^2\sin^2\alpha}\right]\operatorname{tg}\alpha. \quad (42)$$

Let us now determine the values of shock waves [5] for real gas parameters.

The change in the velocity of a shock wave is

$$\Delta w = w_{1n} - w_{2n} = w_{1n}\left(1 - \frac{w_{2n}}{w_{1n}}\right). \quad (43)$$

On the basis of expressions (3), (20), (21), (39), upon a number of transformations, expression (43) may be presented thus:

$$\Delta w = w_1\left\{\sin\alpha - \frac{1}{M_1^2\sin\alpha}\frac{2}{k+1}\frac{\xi_{cr}^2}{y_1^2}\right.$$

$$\times\left[1 - \frac{x-1}{x}\frac{1}{\bar{\mu}_T}\,.\,(z_0 - z_1)_{T_1}\right] - \frac{x-1}{k+1}$$

$$\left.\times\frac{k}{x}\frac{\xi_{cr}^2}{\bar{\mu}_T}\sin\alpha\right\}. \quad (44)$$

The change in a pressure in a shock wave is

$$\Delta p = p_2 - p_1.$$

With regard for expressions (14), (15), (42) we have

$$\Delta p = \rho_1 w_{1n}^2\left\{1 - \frac{1}{M_1^2\sin^2\alpha}\frac{2}{k+1}\frac{\xi_{cr}^2}{y_1^2}\right.$$

$$\times\left[1 - \frac{x-1}{x}\frac{1}{\bar{\mu}_T}\,.\,(z_0 - z_1)_{T_1}\right]$$

$$\left.-\frac{x-1}{k+1}\frac{k}{x}\frac{\xi_{cr}^2}{\bar{\mu}_T}\right\}k. \quad (45)$$

The change in the density in a shock wave is

$$\Delta\rho = \rho_2 - \rho_1$$

or with regard for expression (15) it is possible to write

$$\Delta\rho = \rho_1\left(\frac{w_{1n}}{w_{2n}} - 1\right). \tag{46}$$

Proceeding from the simultaneous soluuon of equations (42) and (44) after determining w_{1n}/w_{2n} and substituting it into expression (46) we arrive at

$$\Delta\rho = \rho_1\left\{1 - \frac{1}{M_1^2\sin^2\alpha}\cdot\frac{2}{k+1}\frac{\xi_{cr}^2}{y_1^2}\left[1 - \frac{x-1}{x}\right.\right.$$
$$\left.\left.\times\frac{1}{\bar{\mu}_T}(z_0 - z)_{T_1}\right] - \frac{x-1}{k+1}\frac{k}{x}\frac{\xi_{cr}^2}{\bar{\mu}_T}\right\}$$
$$\times\left\{\frac{1}{M_1^2\sin^2\alpha}\frac{2}{k+1}\frac{\xi_{cr}^2}{y_1^2}\left[1 - \frac{x-1}{x}\frac{1}{\bar{\mu}_T}\right.\right.$$
$$\left.\left.\times(z_0 - z_1)_{T_1}\right] + \frac{x-1}{k+1}\frac{k}{x}\frac{\xi_{cr}^2}{\bar{\mu}_T}\right\}^{-1}. \tag{47}$$

The change in the temperature in a shock wave is

$$\Delta T = T_2 - T_1 = \frac{1}{1 + b_{\text{mean}}}\frac{(x-1)k}{2x}y_1^2T_1M_1^2$$
$$\times\sin^2\alpha.\left[\left[1 - \left\{\frac{1}{M_1^2\sin\alpha}\frac{2}{k+1}\frac{\xi_{cr}^2}{y_1^2}\right.\right.\right.$$
$$\times\left[1 - \frac{x-1}{x}\frac{1}{\bar{\mu}_T}(z_0 - z_1)_{T_1}\right]$$
$$\left.\left.\left.+\frac{x-1}{k+1}\frac{k}{x}\frac{\xi_{cr}^2}{\bar{\mu}_T}\right\}^2\right]\right]. \tag{48}$$

III. METHODS OF USE OF RELATIONS OBTAINED AND SEQUENCE OF CALCULATION

Assume that the problem on determination of values of shock waves for different parameters expressed by relations (44), (45), (47) and (48) is stated. The initial quantities, i.e. w_1, p_1, T_1 as well as the angle α and the index of the adiabatic curve of a real gas k are prescribed.

Let us determine different quantities entering into equation (44).

According to equation (44) write down

$$M_1^2 = \frac{w_1^2}{a_1^2} = \frac{w_1^2}{y_1^2kRT_1} \tag{49}$$

where

$$y_1^2 = z_1^2\left\{k\left[(\mu_T)_1 - \frac{x-1}{x}(\mu_p)_1\right]\right\}^{-1}. \tag{50}$$

The coefficient z_1 is determined from the gas compressibility diagram by p_1 and T_1, and the coefficients $(\mu_T)_1$ and $(\mu_p)_1$ by expressions (7). If there are no data on c_p, then it is possible to take $x = k$ with a sufficient accuracy. Partial derivatives entering into expression (7) are found by the graphical differentiation method, the method presented in [7] (Fig. 2), being successful.

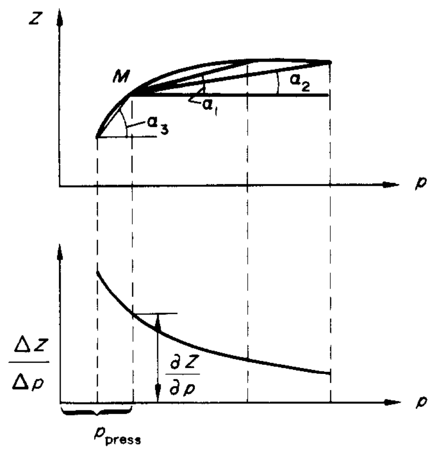

FIG. 2.

The expression for $\bar{\mu}_T$ entering into expression (44) is found as

$$\bar{\mu}_T = \tfrac{1}{2}[(\mu_T)_0 + (\bar{\mu}_T)_{1-2}].$$

Bearing in mind that

$$\mu_T = z - p\left(\frac{\partial z}{\partial p}\right)_T$$

in this case the quantities $(\mu_T)_0$ and $(\bar{\mu}_T)_{1-2}$ are respectively found as

$$(\mu_T)_0 = z_0 - p_0\left(\frac{\partial z}{\partial p}\right)_T \tag{51}$$

$$(\bar{\mu}_T)_{1-2} = \tfrac{1}{2}[(\mu_T)_1 + (\mu_T)_2]. \tag{52}$$

To determine the coefficients μ_T and μ_p it is possible to use also the data from [8] where μ_T and μ_p are designated through z_p and z_T, respectively. These functions presented in [8] with a reference to W. C. Edmister's works and related to an eccentricity factor are given by R. Reid and Volbert in the form

$$z_T = z_T^0 + \omega z_T' \tag{53}$$

$$z_p = z_p^0 + \omega z_p'. \tag{54}$$

Numerical values of ω are given for different gases in Appendix to [8], and the functions z_T^0, z_T', z_p^0 and z_p' depending on a reduced temperature and a reduced pressure are presented in the appropriate diagrams and tables [8].

In order to calculate $(\mu_T)_0$ and $(\bar{\mu}_T)_{1-2}$ it is necessary to know the stagnation parameters after a shock wave (parameters before a shock wave are prescribed). With a sufficient accuracy these quantities may be taken for an ideal gas flow [3]
where

$$M_{1\,\mathrm{ideal}}^2 = \frac{w_1}{kRT_1} \tag{55}$$

$$(p_2)_{\mathrm{ideal}} = p_1\frac{k-1}{k+1}\left(\frac{2k}{k-1}M_{1\,\mathrm{ideal}}^2\sin^2\alpha - 1\right) \tag{56}$$

$$(T_2)_{\mathrm{ideal}} = T_1\left[\left(\frac{k-1}{k+1}\right)^2\left(\frac{2k}{k-1}\cdot M_{1\,\mathrm{ideal}}^2 \times \sin^2\alpha - \right)\cdot\left(\frac{2}{k-1} \times \frac{1}{M_{1\,\mathrm{ideal}}^2\cdot\sin^2\alpha} + 1\right)\right]. \tag{57}$$

Using expressions (53)–(57), the value of $\bar{\mu}_T$ is found by equation (21).

The difference $(z_0 - z_1)_{T_1}$ is found in the following way. From the compressibility diagram the value of z_0 is found by $(p_0)_{\mathrm{ideal}}$ and T_1, and the value of z_1, by p_1 and T_1.

The coefficient ξ_{cr}^2 also entering into equation (44) is determined by formula (41).

According to equations (7) and (8) the coefficients $(\mu_T)_{\mathrm{cr}}$ are expressed in the form:

$$\left.\begin{aligned}(\mu_T)_{\mathrm{cr}} &= z_{\mathrm{cr}} - p_{\mathrm{cr}}\left(\frac{\partial z}{\partial p}\right)_T \\ (\mu_p)_{\mathrm{cr}} &= z_{\mathrm{cr}} + T_{\mathrm{cr}}\left(\frac{\partial z}{\partial T}\right)_p.\end{aligned}\right\} \tag{58}$$

In order to find these quantities it is necessary to have the values of critical flow parameters, p_{cr} and T_{cr}, which may be found from the expressions

$$\begin{aligned}p_{\mathrm{cr}} &= p_0\left(\frac{2}{\bar{n}+1}\right)^{\frac{\bar{n}}{\bar{n}+1}} \\ T_{\mathrm{cr}} &= T_0\cdot\frac{2}{\bar{n}+1}\end{aligned} \tag{59}$$

After determining Δw_1 according to expression (44) it is easy to evaluate a change of a pressure in a shock wave, i.e. Δ_p by formula (45), in this case

$$\rho_1 = \frac{p_1}{z_1 R T_1}.$$

The change in a density $\Delta\rho$ and a temperature ΔT in a shock wave is found by formulae (47) and (48).

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QUELQUES RELATIONS POUR LES ECOULEMENTS SOUS HAUTE PRESSION AVEC OU SANS TRANSFERT DE CHALEUR

Résumé—On a fait dans cet article l'analyse des ondes de choc obliques dans des écoulements de gaz réels. On a obtenu des expressions qui décrivent le comportement des paramètres tels que la pression et les températures dans une onde de choc. Les expressions différent de celles conventionnelles par le fait que la description mathématique des effets dans une onde de choc implique différents coefficients et quantités caractéristiques d'un gaz réel.

On présente une méthode de détermination des facteurs de correction avant et après une onde de choc. Une expression est dérivée pour un angle de déflection. On obtient ensuite des expressions analytiques qui définissent les discontinuités de pression, densité et température.

On décrit une procédure de calcul.

BEZIEHUNGEN FÜR STRÖMUNGEN BEI HOHEN DRÜCKEN MIT UND OHNE WÄRMEÜBERGANG

Zusammenfassung—In einem strömenden, realen Gas werden schräge Stosswellen untersucht. Man erhält Ausdrücke, die das Verhalten von Parametern, wie Druck und Temperatur in einer Stosswelle beschreiben. Die Ausdrücke unterscheiden sich von den bisherigen darin, dass die vorliegende mathematische Beschreibung der Effekte in einer Stosswelle verschiedene Koeffizienten und Grössen umfasst, die auf reales Gas zutreffen.

Eine Methode zur Bestimmung von Korrekturfaktoren vor und hinter einer Stoffwelle wird angegeben. Ein Ausdruck für den Stossfrontwinkel wird abgeleitet. Dann erhält man analytische Ausdrücke, die Druck-, Dichte- und Temperatursprünge bestimmen.

Ein Rechenbeispiel wird ebenfalls gegeben.

НЕКОТОРЫЕ ЗАВИСИМОСТИ ДЛЯ ПОТОКОВ ВЫСОКИХ ДАВЛЕНИЙ ПРИ НАЛИЧИИ И ОТСУТСТВИИ ТЕПЛООБМЕНА

Аннотация—В работе проводится теоретическое исследование косых скачков уплотнения в течении реального газа. Получены развернутые выражения, характеризующие изменения параметров, а именно плотностей, давлений и температур в скачке. Эти выражения отличаются от обычно применяемых тем, что в предполагаемом математическом описании явлений скачков фигурируют различные коэффициенты и величины, характеризующие свойства реального в термодинамическом смвсле газа.

Дан способ определения значений корректирующих коэффициентов до и после скачка. Выведено выражение для угла отклонения потока. Затем получены аналитические выражения, определяющие величины скачков плотности и температуры в потоке реального газа.

В заключение дана методика использования и последовательность расчета полученных зависимостей.